

# MICROSTRIP ANTENNA FOR WIRELESS LAN APPLICATIONS BY APPLYING MODIFIED SMITH-CHART REPRESENTATION

**Settapong Malisuwan, Ph.D.**

**Department of Electrical Engineering  
Chulachomklao Royal Military Academy  
Nakhon-Nayok, Thailand**

**Monton Charoenwattanaporn & Ut Goenchanart, Ph.D.**

**Department of Electrical Engineering  
MET Program, Rangsit University  
Pathumtani, Thailand**

**Vichate Ungvichian, Ph.D.**

**Department of Electrical Engineering  
Florida Atlantic University  
Boca Raton, FL 33432, USA.**

## **Abstract**

*The proposed algorithm depicting the dynamic permittivity of the microstrip structure directly leads to a convenient and modified Smith-chart representation that includes the frequency-dependent influence of fringing field and the lossy characteristics cohesively. The efficacy of the model is illustrated with an example concerning a microstrip patch antenna for wireless LAN applications. Relevant simulations show that the input impedances calculated from the model are more accurate than those from the previous model in the literature by comparing to the measure results, as illustrated with an example of a patch*

*antenna. This model is compatible for CAD efforts with MATLAB™ facilitating fast and user-friendly implementations.*

**Key word:** CAD, Smith chart, Resonant frequency, Rectangular microstrip antenna and Wireless LAN.

## **1 INTRODUCTION**

The microstrip antenna is one of the most popular types of antennas for wireless LAN applications, because it is easily integrated with other passive and active microwave devices. Relevant design equations in closed-form using semi-

empirical strategies specifying the frequency-dependent effective permittivity concept and dispersion characteristics of a microstrip line have been derived in the existing literature [1]-[7]. Although many computer-aided design (CAD) systems have been developed using such algorithms with built-in microstrip design capabilities, simple calculation methods for microstrip line parameters by hand-calculator and/or by personal computer are needed for preliminary design purposes, and/or for quick circuit evaluation purposes. Moreover, designers may need to observe the physical considerations of microstrip circuits on step-by-step basis. Therefore, many researchers are in search of simple methods, which are at the same time sufficient to explain the physical aspects of microstrip circuits, precisely.

The Smith-chart is a powerful visualization tool used in high-frequency engineering for designing impedance-matching circuits, filters, amplifiers, and evaluating the transmission line characteristics. Although there are other impedance and reflection coefficient charts that can be used for such problem, the Smith-chart is probably the best known and most widely used. It was developed in 1939 by P. Smith at the Bell Telephone Laboratories. Today, it has become an integral part of much of the current computer-aided design (CAD) software for high-frequency designs.

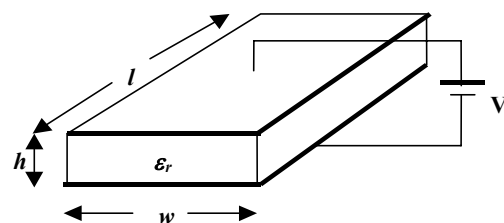
In this research, the proposed algorithm depicting the dynamic permittivity of the microstrip structure directly leads to a convenient and modified Smith-chart representation that includes the frequency-dependent influence of fringing field and the lossy characteristics cohesively. Results based on the proposed model are compared with the available data in the literature in respect to a microstrip patch antenna in the frequency range of wireless LAN

applications. The proposed model is set for use in computer-aided microstrip design and is fully compatible with the needs and trends of modern computer-aided microstrip antenna design.

## 2 FREQUENCY-DEPENDENT SMITH-CHART (MODIFIED SMITH-CHART)

The Smith-chart is an impedance representation in a complex plane depicting a set of circles of constant resistance and partial circles of constant reactance. The standard Smith-chart is based on the static characteristic impedance ( $Z_0$ ) and does not include the frequency-dependent aspects of  $Z_0$ . Therefore, a frequency-dependent factor is necessary for inclusion in the calculations so as to improve the accuracy of the models.

In this section, the frequency-dependent effective permittivity concept is applied to construct a frequency-dependent (lossy) Smith-chart to analyze microstrip line characteristics. Before deriving the frequency-dependent Smith-chart relations, the capacitance parameter in microstrip-line system can be considered. The classical parallel-plate capacitor is shown in Fig. 1. From the geometry shown in Fig. 1, the capacitance per unit length of the structure can be expressed as [8]:



**Fig. 1:** A parallel-plate capacitor

$$C = \varepsilon \frac{w}{h} \quad (1)$$

$$f_n = \frac{fh}{10^6} \quad (9)$$

A simple frequency-dependent capacitance of the parallel-plate capacitor can be modeled in terms of any frequency-dependent attributes of  $\varepsilon$ . That is,

$$C(\omega) = \varepsilon_o \varepsilon(\omega) \frac{w}{h} \quad (2)$$

Where  $\varepsilon(\omega)$  is a complex permittivity equal to

$$\varepsilon_r - \frac{\varepsilon_r - \varepsilon_{eff}(0)}{1 + Q(\omega)} \quad (3)$$

$Q(\omega)$  is the frequency-dependent term given by [18]:

$$Q(\omega) = P_1 P_2 [(0.1844 + P_3 P_4) f_n]^{1.5763} \quad (4)$$

$$P_1 = 0.27488 + u^{0.6315} + \frac{0.525}{(1 + 0.0157 f_n)^{20}} - 0.065683 \exp(-8.7513u) \quad (5)$$

$$P_2 = 0.33622 [1 - \exp(-0.03442 \varepsilon_r)] \quad (6)$$

$$P_3 = 0.0363 \exp(-4.6u) \left[ 1 - \exp\left(-\frac{f_n}{38.7}\right)^{4.97} \right] \quad (7)$$

$$P_4 = 1 + 2.75 \left[ 1 - \exp\left(-\frac{\varepsilon_r}{15.916}\right)^8 \right] \quad (8)$$

Where  $f_n$  GHz.mm is the frequency normalized with respect to the substrate height,

Therefore,

$$C(\omega) = \varepsilon_o \varepsilon_r \frac{w}{h} - \frac{\varepsilon_r - \varepsilon_{eff}(0)}{1 + Q(\omega)} \frac{w}{h} = \varepsilon_o \varepsilon_r \frac{w}{h} \left[ 1 - \frac{1 - \varepsilon_{eff}(0)/\varepsilon_r}{1 + Q(\omega)} \right] \quad (10)$$

$$C(\omega) = C \left[ 1 - \frac{1 - \varepsilon_{eff}(0)/\varepsilon_r}{1 + Q(\omega)} \right] \quad (11)$$

where  $C = \varepsilon_o \varepsilon_r (w/h)$ .

For simplicity, the coefficients of Eqn. (11) are defined as follows:

$$b = \left[ 1 - \frac{1 - \varepsilon_{eff}(0)/\varepsilon_r}{1 + Q(\omega)} \right] \quad (12)$$

If  $G$  (conductance per unit length) and  $R$  (resistor per unit length) are neglected, the characteristic impedance can be written as:

$$Z_0 = \sqrt{\frac{L}{C}} \quad (13)$$

To obtain the frequency-dependent characteristic impedance ( $Z_0'(\omega)$ ), the frequency-dependent capacitance ( $C(\omega)$ ) of Eqn. (11) is substituted into the capacitance ( $C$ ) in Eqn. (13). The resulting frequency-dependent characteristic impedance is then given by:

$$Z_0'(\omega) = \sqrt{\frac{L}{C(\omega)}} = \sqrt{\frac{L}{Cb}} = \frac{Z_0}{\sqrt{b}} \quad (14)$$

Now, the frequency-dependent (lossy) Smith-chart can be constructed by applying  $Z_0'(\omega)$  in Eqn.(14) into the normalized terminal impedance expression following the procedure as

that for a standard Smith-chart [9]. Hence, the resulting normalized terminal impedance  $z'_L$  is given by

$$z'_L = \frac{Z_L}{Z_0} = br + jbx \quad \text{(Dimensionless)} \quad (15)$$

where r and x are the normalized resistance and normalized reactance, respectively.

Corresponding, the voltage reflection coefficient of present Smith chart can be expressed as:

$$\Gamma = \Gamma_r + j\Gamma_i = \frac{z'_L - 1}{z'_L + 1} \quad (16)$$

or

$$z'_L = \frac{Z_L}{Z_0} = br + jbx = \frac{(1 + \Gamma) + j\Gamma_i}{(1 - \Gamma) - j\Gamma_i} \quad (17)$$

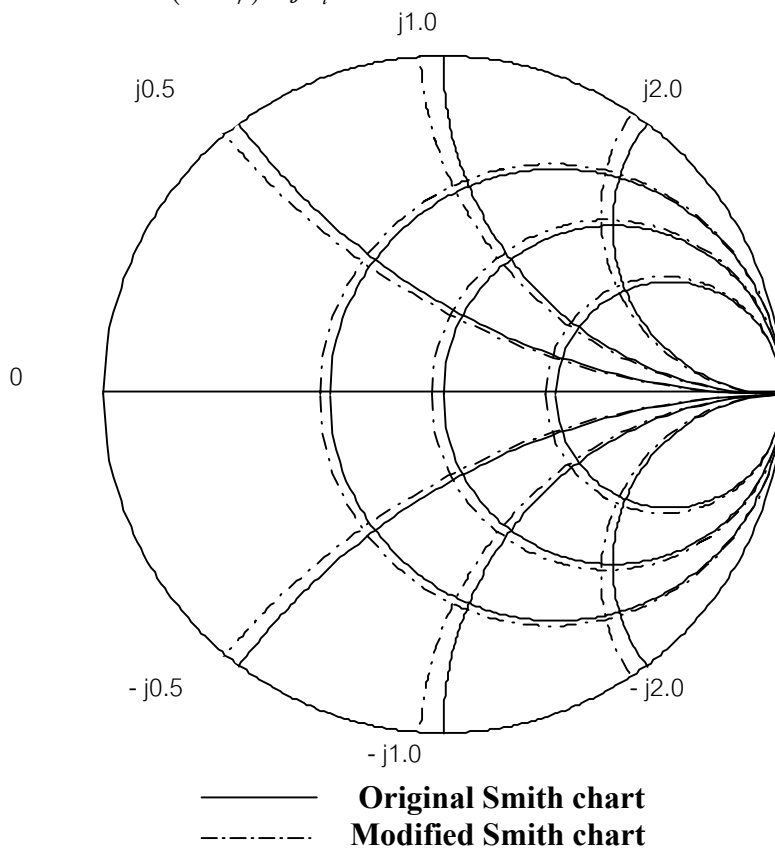
Now, the desired set of equations depicting the modified Smith-chart are:

$$\Gamma_r^2 - \frac{br}{1 + br} + \Gamma_i^2 = \frac{1}{(1 + br)^2} \quad (18)$$

and

$$(\Gamma_r - 1)^2 + \Gamma_i^2 - \frac{1}{bx} = \frac{1}{bx^2} \quad (19)$$

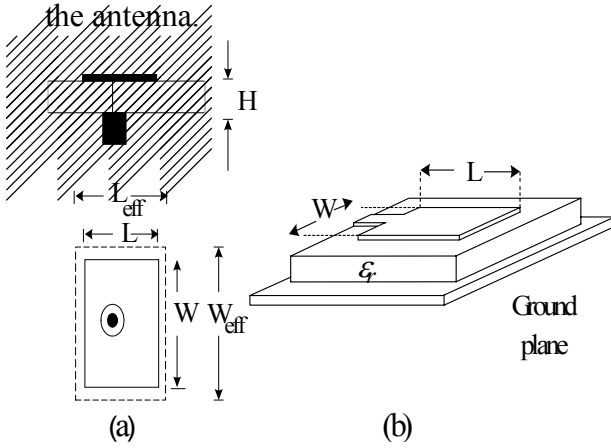
An example of Modified Smith-chart is given in Fig. 2. This figure is a comparison of an original Smith-chart and modified Smith-chart, which calculated from the geometry of a microstrip antenna for a wireless LAN system with operating frequency at 2.45 GHz. Fig.2 shows that the effect of dynamic permittivity will lead to a modification of the Smith chart.



**Fig. 2:** Original Smith chart compared with modified Smith chart for coax-fed rectangular microstrip patch antenna operating at 2.45 GHz,  $\epsilon_r=4.53$ ,  $Z_0=50\ \Omega$ ,  $H=0.300\text{cm}$ ,  $L=2.78\text{cm}$ ,  $W=2.85\text{cm}$ .

### 3 INPUT IMPEDANCE OF THE MICROSTRIP PATCH ANTENNA

A microstrip antenna may be excited or 'fed' by different types of transmission lines, for example coaxial, microstrip, or coplanar. Two different types of feed are shown in Figs. 3 (a) and (b). The radiating elements may be fed directly, with electrical continuity between the conductor of the transmission line and the conducting patch. On the other hand, the microstrip patch antenna fed by a transmission line behaves as a complex impedance  $Z_{in} = (R + jX)$ , which depends mainly on the geometry of the coupling between the transmission line and the antenna.



**Fig. 3:** Geometry of a rectangular microstrip patch antenna (a) coax feed. (b) direct feed

The input impedance of the structure shown in Fig. 3 is given by [10]:

$$Z(f) = \frac{R}{1 + Q_T^2 \left( \frac{f}{f_R} - \frac{f_R}{f} \right)^2} + j^2 X_L - \frac{R Q_T \left( \frac{f}{f_R} - \frac{f_R}{f} \right)^2}{1 + Q_T^2 \left( \frac{f}{f_R} - \frac{f_R}{f} \right)^2} \quad (20)$$

where  $R$  is the resonant resistance including the influence of the fringing field at the edges of the patch;  $f$  is the operating

frequency; and  $f_R$  is the resonant frequency. It can be written as [10]:

$$R = \frac{Q_T H}{\pi f_R \epsilon_{dyn} \epsilon_0 L W} \cos^2 \left( \frac{\pi X_0}{L} \right) \quad (21)$$

where  $\epsilon_{dyn}$  is the dynamic permittivity.

$Q_T$  is the quality factor associated with system losses, which include radiation from the wall ( $Q_R$ ), Losses in the dielectric ( $Q_D$ ) and Losses in the conductor ( $Q_C$ ).  $Q_T$  is calculated by [10]:

$$Q_T = \frac{1}{Q_R} + \frac{1}{Q_C} + \frac{1}{Q_D}^{-1} \quad (22)$$

where  $Q_R$ ,  $Q_D$  and  $Q_C$  are given below

$$Q_R = \frac{c_0 \sqrt{\epsilon_{dyn}}}{4 f_R H} \quad (23)$$

$$Q_D = \frac{1}{Tg\delta} \quad (24)$$

$$Q_C = \frac{0.786 \sqrt{f_R Z_{ao}(W) H}}{P_a} \text{ for copper, } f_R \text{ in GHz} \quad (25)$$

where  $Z_{ao}(W)$  is the impedance of an air filled microstrip line of width ( $W$ ) and thickness ( $H$ ).  $Z_{ao}(W)$  is evaluated by setting  $\epsilon_r=1$ . The impedance of a dielectric filled line can be written as [10]:

$$Z_a(W) = \frac{60\pi \frac{W}{2H}}{\sqrt{\epsilon_r}} + 0.441 + 0.082 \frac{\epsilon_r - 1}{\epsilon_r} + \frac{(\epsilon_r + 1)}{2\pi\epsilon_r} \left[ 1.451 + Ln \left( \frac{W}{2H} \right) + 0.94 \frac{W}{2H} \right]^{-1}, \quad W/H > 1$$

(26)

$$C_{dyn}(\epsilon) = C_{0,dyn}(\epsilon) + 2C_{e1,dyn}(\epsilon) + 2C_{e2,dyn}(\epsilon) \quad (31)$$

and

$$P_a = \frac{2\pi \frac{W}{H} + \frac{W/(\pi H)}{W/(2H) + 0.94} + \frac{H}{W}}{\frac{W}{H} + \frac{2}{\pi} \ln \frac{W}{2H} + 0.94} \quad (27)$$

To take the effect of coax-feed probe (Fig. 3) into account, it is necessary to modify the input impedance by an inductive reactance term [11], given by

$$X_L = \frac{377 fH}{c_0} \ln \frac{c_0}{\pi d_0 \sqrt{\epsilon_r}} \quad (28)$$

where  $c_0$  is the velocity of light in vacuum and  $d_0$  is the diameter of the probe.

$$f_R = f_{mn} = \frac{c_0}{2\sqrt{\epsilon_{dyn}}} \sqrt{\frac{m^2}{W_{eff}^2} + \frac{n^2}{L_{eff}^2}} \quad (29)$$

where  $W_{eff}$  is the effective width,  $L_{eff}$  is the effective length and  $\epsilon_{dyn}$  is the dynamic permittivity which is a function of dimension (W, L, H) [10]:

$$\epsilon_{dyn} = C_{dyn}(\epsilon) / C_{dyn}(\epsilon_0) \quad (30)$$

where  $C_{dyn}(\epsilon)$  represents the total dynamic capacitance of the patch in the presence of a dielectric of relative permittivity and  $C_{dyn}(\epsilon_0)$  represents the total dynamic capacitance of the patch in the presence of air,  $C_{dyn}(\epsilon)$  can be written as

where  $C_{0,dyn}(\epsilon)$  is the dynamic main field of the patch capacitance without considering the fringing field.  $C_{e,dyn}(\epsilon)$  is the dynamic edge field of the patch capacitance with considering the fringing field for each side of the patch, which can be expressed by [10]:

$$C_{0,dyn}(\epsilon) = \frac{\epsilon_0 \epsilon_r WL}{H \gamma_n \gamma_m} = \frac{C_{0,stat}(\epsilon)}{\gamma_n \gamma_m} \quad (32)$$

where  $C_{0,stat}(\epsilon)$  represents the static main capacitance of the patch without considering the fringing field and  $\gamma_n$  and  $\gamma_m$  are in the form:

$$\gamma_i = \begin{cases} 1 & \text{for } i = 0 \\ 2 & \text{for } i \neq 0 \end{cases} \quad (33)$$

Assuming that the edge-field of the resonator has an x- and y- dependent field distribution, the dynamic fringing capacitances are then given in the general form by [10]:

$$C_{e1,dyn}(\epsilon) = \frac{1}{\gamma_n} \frac{Z(W, H, \epsilon_r = 1)}{c_0 Z^2(W, H, \epsilon_r)} - \frac{\epsilon_0 \epsilon_r W}{H} \quad (34)$$

$$C_{e2,dyn}(\epsilon) = \frac{1}{\gamma_m} \frac{Z(L, H, \epsilon_r = 1)}{c_0 Z^2(L, H, \epsilon_r)} - \frac{\epsilon_0 \epsilon_r L}{H} \quad (35)$$

where  $Z(W, H, \epsilon_r)$  is the characteristic impedance of the microstrip line [12], [13].

If the effect of the strip thickness is neglected,

$$Z(W, H, \epsilon_r) = \frac{377}{\sqrt{\epsilon_{eff}(W)}} \left( \frac{W}{H} + 1.393 + 0.667Ln\left(\frac{W}{H}\right) + 1.444\left(\frac{W}{H}\right)^{-1} \right) \quad (36)$$

However, in the proposed model, more accurate expression for the characteristic impedance [3] is considered. It is given by:

$$Z(W, H, \epsilon_r) = \frac{377}{2\pi} Ln\left(\frac{f(W/H)}{(W/H)}\right) + \sqrt{1 + \frac{2}{(W/H)}} \quad (37)$$

$$f(W/H) = 6 + (2\pi - 6) \exp\left(-\frac{30.666}{(W/H)}\right)^{0.7528} \quad (38)$$

To evaluate  $\epsilon_{eff}$ , the following simple equation from [14] is adopted [10]:

$$\epsilon_{eff}(W) = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(1 + \frac{10}{W/H}\right)^{-1/2} \quad (39)$$

To obtain  $C_{dyn}(\epsilon_0)$ ,  $\epsilon$  can be replaced by  $\epsilon_0$  in all of the above equations.

To take the effect of the fringing field at the corners and the dielectric inhomogeneity [10] of the rectangular microstrip patch antenna, the  $W_{eff}$  and  $L_{eff}$  can be calculated from the following relation:

$$L_{eff} = L + \frac{W_{eq} - W}{2} \frac{\epsilon_{eff}(W) + 0.3}{\epsilon_{eff}(W) - 0.258} \quad (40)$$

where  $W_{eq}$  is the equivalent width

$$W_{eq} = \frac{120\pi H}{Z_a(W)\sqrt{\epsilon_{eff}(W)}} \quad (41)$$

Similarly, we can calculate  $W_{eff}$  from equation (40) and (41) by replacing  $L_{eff}$ ,  $L$ ,  $W_{eq}$ ,  $W$  with  $W_{eff}$ ,  $W$ ,  $L_{eq}$ ,  $L$  respectively.

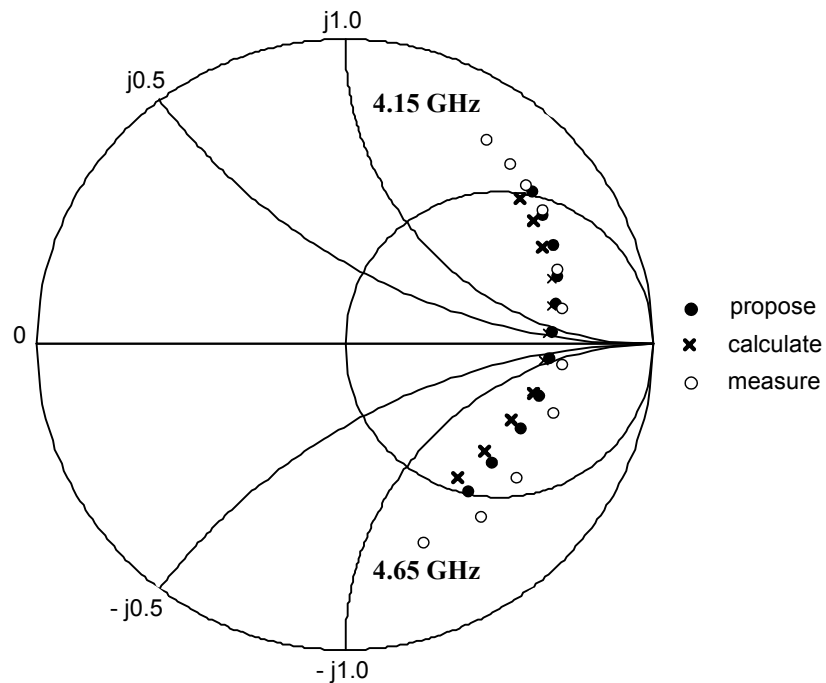
#### 4 ANTENNA SYSTEM IN WIRELESS COMMUNICATION APPLICATIONS (ISM BAND): DESIGN CONSIDERATIONS

In this section, the applications of the modified Smith-chart on the rectangular microstrip patch antenna design are illustrated. The results of modified Smith-chart model are compared with the results by using the method in [10], [16]. They are also compared with measured values [15] concerning the fundamental mode ( $m = 0$ ,  $n = 1$ ). In Table 1, the results on resonant frequency are presented. It can be seen that the results of the proposed model are better than those predicted by [10] and [16] and are in good agreement with experiment [15].

Figs. 4 and 5 show the input impedance for a patch antenna operating at 4.5 GHz and 3.7 GHz, respectively. The proposed model results are compared with the computed results in [10] and measured data of [12]. The results indicate that the proposed model gives results close to the experimental data. It can also be observed that the present results are better than those predicted in [10]. The reason is that, in the proposed model, the frequency-dependent characteristic impedance is more comprehensively addressed included in the algorithm so that possible errors in the high frequency are reduced.

**Table 1.** Comparisons of measured and calculated resonant frequency of a rectangular microstrip patch antenna ( $\epsilon_r=2.33$ )

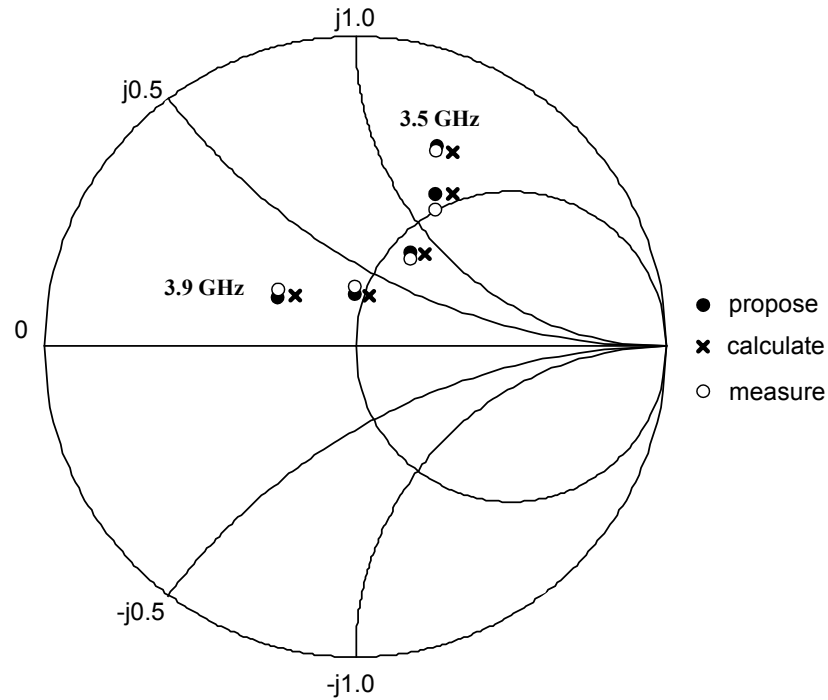
Dimension (cm)			Resonant Frequency ( $f_R$ ,GHz)			
W	L	H	Measure [15]	James [16]	Abboud [10]	Model
5.70	3.80	0.32	2.31	2.30	2.38	2.37
4.55	3.05	0.32	2.89	2.79	2.91	2.90
2.95	1.95	0.32	4.24	4.11	4.29	4.28
1.95	1.30	0.32	5.84	5.70	5.96	5.96
1.70	1.10	0.32	6.80	6.47	6.76	6.77
1.40	0.90	0.32	7.70	7.46	7.82	7.84
1.20	0.80	0.32	8.27	8.13	8.50	8.54
1.05	0.70	0.32	9.14	8.89	9.30	9.34
0.90	0.60	0.32	10.25	9.82	10.27	10.32
1.70	1.10	0.15	7.87	7.46	7.79	7.75
1.70	1.10	0.32	6.80	6.47	6.76	6.77
1.70	1.10	0.95	4.73	4.32	4.52	4.53



**Fig. 4:** Input impedances of coax-fed rectangular microstrip patch antenna



$\epsilon_r=2.55, Tg\sigma=0.002, H=0.159\text{cm}, d_0=0.127\text{cm}, Z_0=50\Omega$ , mode  $(m=0,n=1)$ ,  $L=2.01\text{cm}$ ,  
 $W=2.01\text{cm}, X_0=0.13\text{cm}$ .



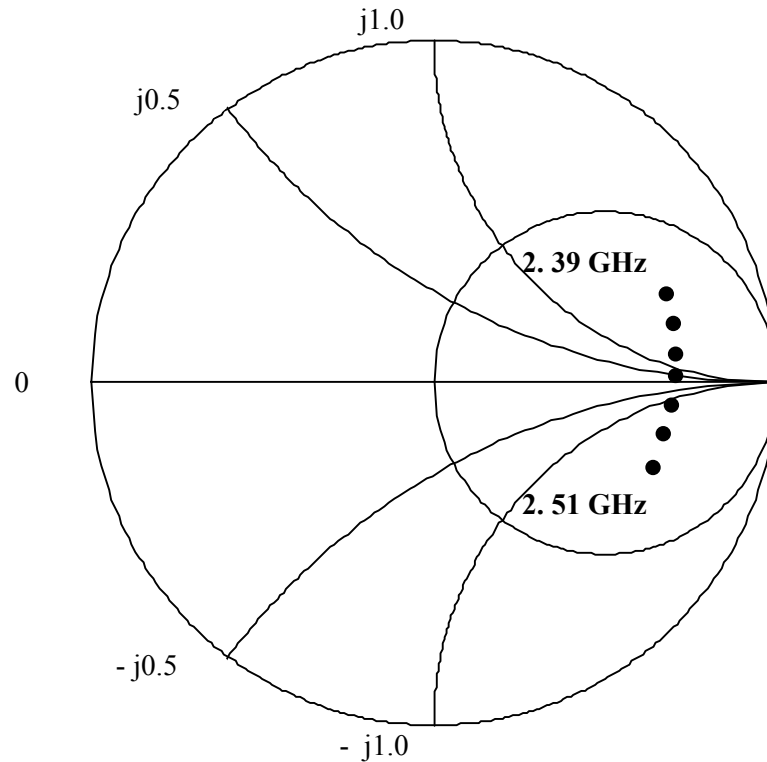
**Fig. 5:** Input impedances of coax-fed rectangular microstrip patch antenna  
 $\epsilon_r=4.53, Tg\sigma=0.025, H=0.300\text{cm}, d_0=0.065\text{cm}, Z_0=50\Omega$ , mode  $(m=0,n=1)$ ,  $L=1.74\text{cm}$ ,  
 $W=2.31\text{cm}, X_0=0.55\text{cm}$ .

## 5 DESIGN OF MICROSTRIP ANTENNA FOR A WIRELESS LAN APPLICATION BY USING MODIFIED SMITH-CHART REPRESENTATION

To design a microstrip antenna for wireless LAN applications, designers must know an operating frequency of wireless LAN systems. In the United States, the Federal Communications Commission (FCC) governs radio transmissions, including those employed in wireless LANs. Other nations have corresponding regulatory agencies. Wireless LANs are typically designed to operate in portions of the radio spectrum where the FCC does not require the end-user to purchase license to use the airwaves. In the U.S. most wireless LANs broadcast over one of the ISM (Instrumentation, Scientific,

and Medical) bands. These include 902-928 MHz, 2.4-2.483 GHz, 5.15-5.35 GHz, and 5.725-5.875 GHz. For wireless LANs to be sold in a particular country, the manufacturer of the wireless LAN must ensure its certification by the appropriate agency in that country.

In this section, the proposed Smith-chart is utilized to design a rectangular microstrip antenna for wireless LAN applications. For the present design, the rectangular microstrip antenna has a substrate with dielectric constant ( $\epsilon_r$ ) of 4.53 and the antenna is a coax-fed type. The size of the patch is 2.85 cm ( $w$ )  $\times$  2.78 cm ( $l$ ) (at  $f=2.45$  GHz) and a thickness of  $h=0.3$  cm. Fig. 6 shows the input impedance for this patch antenna model.



**Fig. 6:** Input impedances of coax-fed rectangular microstrip patch antenna operating at 2.45 GHz  $\epsilon_r=4.53, Tg\sigma=0.002, H=0.300\text{cm}, d_0=0.5573\text{cm}, Z_0=50\Omega, L=2.78\text{cm}, W=2.85\text{cm}, X_0=0\text{ cm},$  mode (m=0,n=1).

measure, as illustrated with an example of a patch antenna.

## 6 CONCLUDING REMARKS

The use of modified Smith-chart is proved to be a method representing the frequency-dependent characteristics of microstrip antennas for wireless LAN applications. The present study demonstrates the feasibility of a cohesive presentation of the dispersion (lossy and lossless) characteristics of a microstrip line, which is compatible for CAD efforts. Relevant simulations show that the input impedances calculated from the model are more accurate than those from the Abboud's model by comparing to the

In summary, the technique described in this paper offers a strategy for portraying the frequency-dependent characteristics of microstrip antennas via a modified Smith-chart representation for wireless LAN applications.

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