

# Applying Grey Relational Analysis to the Vendor Evaluation Model

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## Abstract

*Selecting a proper vendor to meet production demand is a common problem that most manufacturing enterprises have to face. Normally, manufacturers conduct a performance evaluation on each vendor before a new product comes into mass production. The selection process is based on their previous performance records, so the ranking determines which vendor will get this supply contract. However, a survey on current evaluation methods shows that they are all less objective and lack accurate data processing. For example, most commonly used evaluation methods are based on various characteristics of a good vendor as measurement parameters. Nevertheless, whether the characteristics are quantitative or qualitative, the method uses a rating scale for each characteristic. The numeric score for each characteristic will be weighted by a factor, and then summing all scores together. As a result, achieving the highest grade would imply the best vendor candidate. Therefore, this study will utilize the Grey relational analysis in the Grey*

*theory (Deng, 1982) to establish a complete and accurate evaluation model for selecting vendors. This methodology will significantly reduce the purchasing cost and increase the production efficiency and overall competitiveness.*

**Keywords:** *Grey theory, Grey relational analysis, Selecting vendors*

## 1. Introduction

Any organization in business today is under pressure to stay competitive and make profit. Considerable attention has focused on vendor selection in recent years as more managers have come to realize its potential for contributing to the firm's competition advantage. Selecting the best vendor has become a crucial problem for a company. A good vendor must supply high-quality materials, deliver proper quantities of materials at proper time, reduce costs, and provide excellent services in order to satisfy customers' demands.

It is important to conduct performance evaluation for all vendors either when a company is newly established or before a new product is mass-produced. The selection process is based on their previous performance records, so the ranking determines which vendor will get the supply contract. However, a survey on current evaluation methods, shows they are all less objective and lack accurate data processing. For example, those evaluation methods are based on various characteristics of a good vendor as measurement parameters. Despite the characteristics are quantitative or qualitative, the methods use a rating scale for each characteristic. The numeric score for each characteristic will be weighted by a factor, and then summing all scores together. As a result, achieving the highest grade would imply the best vendor candidate [6, 15]. Hence, the selection of the best vendor in this traditional method is based on various characteristics as evaluation factors, which belongs to multiple attributes decision problems. In the evaluation factors, some are quantitative or qualitative. For example, price is in terms of dollars indicating the quantitative factor. Services are in terms of good, fair, or poor indicating the qualitative factor. Deng [10] had proposed Grey relational analysis in the Grey theory that was already proved to be a simple and accurate method for multiple attributes decision problems [7, 8, 11, 13], especially for those problems with very unique characteristic [3, 5]. Therefore, this study will utilize the Grey relational analysis to establish a complete and accurate evaluation model for selecting vendors. This methodology will significantly reduce the purchasing cost and increase the production efficiency and overall competitiveness.

## 2. Grey Theory

The black box is used to indicate a system lacking interior information [W.R.

Ashby, 1945]. Nowadays, the black is represented, as lack of information, but the white is full of information. Thus, the information that is either incomplete or undetermined, is called Grey. A system having incomplete information is called Grey system. The Grey number in Grey system represents a number with less complete information. The Grey element represents an element with incomplete information. The Grey relation is the relation with incomplete information. Those three terms are the typical symbols and features for Grey system and Grey phenomenon [4]. There are several aspects for the theory of Grey system [4]:

1. Grey generation: This is data processing to supplement information. It is aimed to process those complicate and tedious data to gain a clear rule, which is the whitening of a sequence of numbers.
2. Grey modeling: This is done by step 1 to establish a set of Grey variation equations and Grey differential equations, which is the whitening of the model.
3. Grey prediction: By using the Grey model to conduct a qualitative prediction, this is called the whitening of development.
4. Grey decision: A decision is made under imperfect countermeasure and unclear situation, which is called the whitening of status.
5. Grey relational analysis: Quantify all influences of various factors and their relation, which is called the whitening of factor relation.
6. Grey control: Work on the data of system behavior and look for any rules of behavior development to

predict future's behavior, the prediction value can be fed back into the system in order to control the system.

This study will adopt all six above-mentioned research steps to develop a vendor evaluation model based on Grey relational analysis, and apply to vendor evaluation and selection. All details will be discussed in the following sections. The Grey relational analysis uses information from the Grey system to dynamically compare each factor quantitatively. This approach is based on the level of similarity and variability among all factors to establish their relation. The relational analysis suggests how to make prediction and decision, and generate reports that make suggestions for the vendor selection. This analytical model magnifies and clarifies the Grey relation among all factors. It also provides data to support quantification and comparison analysis [2]. In other words, the Grey relational analysis is a method to analyze the relational grade for discrete sequences. This is unlike the traditional statistics analysis handling the relation between variables. Some of its defects are: (1) it must have plenty of data; (2) data distribution must be typical; (3) a few factors are allowed and can be expressed functionally. But the Grey relational analysis requires less data and can analyze many factors that can overcome the disadvantages of statistics method. The Grey theory and method are described in the following:

### 2.1. Influence space, measurement space, and Grey relational space

Let  $P(X)$  represent the factor set of a specific topics,  $Q$  is the influence relation, then  $\{P(X); Q\}$  is influence space. It must have the following properties [3]:

1. Existence of key factors: for example, the key factors of basketball player are height, weight, and rebound.
2. Numbers of factors are limited and countable: for example each of the height, weight, and rebound are countable.
3. Factor independability: each factor must be independent.
4. Factor expandability: For example, besides the height, weight, and rebound, the free throw attempt can be added as a factor.

The series formed by  $P(X)$  is:

$$x_i^{(0)}(k) = (x_1^{(0)}(1), \dots, x_i^{(0)}(k)) \in X;$$

where  $i = 0, \dots, m$ .  $k = 1, \dots, n$ .  $N$

If the following conditions are satisfied:

1. Nondimension: the numeric value for all factors must be nondimension.
2. Scaling: the factor value for various series must be at the same level.
3. Polarization: if the factor value in the series is described as the same direction, the series is comparable. Then the measurement space is expressed as  $\{P(X); x_i^*(k)\}$ , the Grey relational space formed by the satisfaction of both factor space and comparability is termed by  $\{P(X); \Gamma\}$ .

### 2.2. Generation of Grey relation

Under the principle of series comparability, to achieve the purpose of

Grey relational analysis, we must perform data processing. This processing is called generation of Grey relation or standard processing. The expected goal for each factor is determined by Wu [8] based on the principles of data processing. They are described in the following:

1. If the expectancy is larger-the-better (e.g., the benefit), then it can be expressed by

$$x_{ij} = \frac{X_{ij} - (X_{ij})_{\min}}{(X_{ij})_{\max} - (X_{ij})_{\min}} \quad (1)$$

2. If the expectancy is smaller-the-better (e.g., the cost and defects), then it can be expressed by

$$x_{ij} = \frac{(X_{ij})_{\max} - X_{ij}}{(X_{ij})_{\max} - (X_{ij})_{\min}} \quad (2)$$

3. If the expectancy is nominal-the-best (e.g., the age), and when the targeted value is  $X_o : (X_{ij})_{\max} ? X_o ? (X_{ij})_{\min}$ , then it can be expressed by

$$x_{ij} = \frac{|X_{ij} - X_o|}{(X_{ij})_{\max} - X_o} \quad (3)$$

### 2.3. The Grey relational grade

The measurement formula for quantification in Grey relational space is called the Grey relational grade. When we are determining Grey relation and taking only one series,  $x_0(x)$ , as a referenced series, it is called the grade of local Grey relation. If anyone of the series,  $x_i(x)$ , is referenced series, it is called the grade of global Grey relation. Additionally, the Grey relational coefficient must first be determined before we obtain the Grey relational grade.

In the Grey relational space,  $\{P(X); \Gamma\}$ , there is a series

$$x_i = (x_i(1), x_i(2), \dots, x_i(k), \dots, x_i(n)) \quad X$$

where  $i = 0, \dots, m$ .  $k = 1, \dots, n$ .  $n \geq N$

If the grade of local Grey relation is brought to define the Grey relational coefficient,  $\gamma(x_i(k), x_j(k))$ , it can be expressed as following:

$$\gamma(x_i(k), x_j(k)) = \frac{\Delta_{\min} + \Delta_{\max}}{\Delta_{0i}(k) + \Delta_{\max}} \quad (4)$$

where  $i = 0, \dots, m$ .  $k = 1, \dots, n$ .  $j \neq i$ ;

$x_0$  is a referenced series,  $x_i$  is a specific comparative series;

$\Delta_{0i} = \|x_0(k) - x_i(k)\|$ : representing the  $k$ 's absolute value of the difference of  $x_0$  and  $x_i$ ;

$$\Delta_{\min} = \min_{j \neq i} \min_k \|x_0(k) - x_j(k)\|;$$

$$\Delta_{\max} = \max_{j \neq i} \max_k \|x_0(k) - x_j(k)\|.$$

After obtaining the Grey relational coefficient, we normally take the average of the Grey relational coefficient as the Grey relational grade:

$$\Gamma = \gamma(x_i, x_j) = \frac{1}{n} \sum_{k=1}^n \gamma(x_i(k), x_j(k)) \quad (5)$$

However, since in real application the effect of each factor on the system is not exactly same, Eq. (5) can be modified as:

$$\Gamma = \gamma(x_i, x_j) = \sum_{k=1}^n \beta_k \gamma(x_i(k), x_j(k)) \quad (6)$$

where  $\beta_k$  represents the normalized weighting value of a factor and  $\sum_{k=1}^n \beta_k = 1$  when equating both Eq (5) and (6).

#### 2.4. The Grey relational series

The Grey relational grade represents the correlation between two series. It is not important in a decision-making. Rather, the ranking order of the relational grade is the most important information. Therefore,  $m$ 's comparative series with its corresponding Grey relational grade is rearranged according to the order of the their magnitudes. A Grey relational series is defined as following:

In the Grey relational space,  $\{P(X); \Gamma\}$ , referenced series,  $x_0$ , and comparative series,  $x_i$  and  $x_j$ :

$$\begin{aligned} x_0 &= (x_0(k)), k=1, \dots, n. \\ x_i &= (x_i(k)), k=1, \dots, n; i? I \\ x_j &= (x_j(k)), k=1, \dots, n; j? I \end{aligned}$$

If  $\gamma(x_0, x_i) > \gamma(x_0, x_j)$ , the situation indicating the relational grade of  $x_i$  vs.  $x_0$  is greater than that of  $x_j$  vs.  $x_0$ , or represented by  $\Gamma_{0i} > \Gamma_{0j}$ . This is the relational series for  $x_i$  and  $x_j$  [5].

#### 2.5. Decision for Grey multiple attributes

To solve problems, if many ways or feasible methods exist, we normally make a complete evaluation on those resolutions. Then decision is made based on the evaluation results. It is noted that the multiple attributes decision-making [12] is defined when more than one-evaluation factors are considered. Hence, the application of Grey relational analysis to multiple purposes and attributes is called as Grey multiple attributes decision [5].

Moreover, this method regards each comparative series as a feasible solution, and the numeric score for each evaluation factor becomes the numeric value for each comparative value. The relational grade between comparative series and standard series is then determined. Finally, the decision can be made based on the ranking of each feasible solution. Chang [8] had applied the Grey multiple attributes decision-making to purchase evaluation. He adopted price, function, and quality as evaluation attributes so as to find the best vendor and brand.

### 3. Example for vendor evaluation

This section will concentrate on an example to illustrate how to apply the Grey multiple attributes to the vendor evaluation. Since variations of managerial conditions among vendors must be accounted, and the requirements for their manufacturing process are quite different, it is not our intention to develop an evaluation model with considering all variations as mentioned above. Therefore, this paper will only provide a feasibility study on the new model for vendor evaluation.

#### 3.1. Implementation with evaluation factors and measure parameters

Since the evaluation factors are much dependent on the enterprise environment, the top management of the enterprise may invite the members of the department of purchasing, production control, and quality control to meet together, and decide the appropriate evaluation factors and measure parameters for vendor evaluation. Traditionally, quality, price, delivery date, quantity, and services are chosen to be typical evaluation factors. The measure parameters for these five evaluation factors are shown in Table 1.

Table 1: Evaluation factors and measure parameters for vendor evaluation

Evaluation factors	Quality	Price	Delivery date	Quantity	Services
Measure parameters	Defects	Quotation	Delay rate	Shortage rate	Score
Quantify criteria	$\frac{\text{rejects}}{\text{gross no. of batches}}$	Unit price	$\frac{\text{no. of delays}}{\text{gross no. of ship.}}$	$\frac{\text{no. of short.}}{\text{gross no. of ship.}}$	Five classification levels

Regarding the services of evaluation factors, the corresponding measure parameter can be determined by (1) operated as better methods for defects elimination, and active involvement with vendors and customers; (2) delivery speed; (3) service condition. A qualitative ranking will be given by a review committee, then convert these qualitative ranking into a rating scale of 1 to 5 (larger-the-better). A more subjective overall fuzzy evaluation method may be used to get fuzzy number.

### 3.2. The corresponding weighting value for evaluation factors

Once the evaluation factor has been determined, we are in a position to find the corresponding weighting value for each individual evaluation factor. The weighting value determination can be done by Delphi Method or Eigenvector [14]. Table 2 shows the corresponding weighting value for each evaluation factor.

### 3.3. Implementation with evaluation matrix

It is assumed that five vendors are able to supply certain raw materials. The delivery record is rearranged by purchasing staff as shown in Table 3. Using the data from Table

3, an evaluation matrix can be formed. It is noted that evaluation factor is indicated in attribute column, each vendor is comparative series.

### 3.4. Data rationalizing

The expected goal can be rationalized according to each attribute. A group of assumptions are made for the following: (1) Quality: the quantifying value for reject (smaller-the-better) is 0; (2) Price: the quantifying value for unit price (smaller-the-better) is 0; (3) Delivery date: the quantifying value for delay (smaller-the-better) is 0; (4) Quantities: the quantifying value for shortage (smaller-the-better) is 0; (5) Services: the quantifying value for the service score (larger-the-better) is 5.

The measure value of each attribute is further standardized based on above-mentioned expected goal [9]. The matrix for comparative series is obtained as shown in Table 4.

### 3.5. Establishing standard series

According to our expected goal for each evaluation factor, an ideal standard series ( $X_0 = 1$ ) is established in the last line in Table 4.

Table 2: The corresponding weighting value for each evaluation factor

Evaluation factors	Quality	Price	Delivery date	Quantity	Services
Measure parameter	0.30	0.20	0.15	0.15	0.20

Table 3: Measurement value for each evaluation attribute (delivery data for a period of two years)

Measure Vendors	Quality (Defects)	Price (Unit price)	Delivery date (Delay rate)	Quantity (Shortage rate)	Services (Score)
A	0.15	12	0.15	0.05	2
B	0.22	10	0.25	0.08	4
C	0.15	8	0.15	0.05	5
D	0.08	13	0.30	0.15	4
E	0.12	9	0.05	0.20	3

Table 4: Data rationalizing

Item Comparative series	Quality	Price	Delivery date	Quantity	Services
$X_1$	0.500	0.200	0.600	1.000	0.000
$X_2$	0.000	0.600	0.200	0.800	0.667
$X_3$	0.500	1.000	0.600	1.000	1.000
$X_4$	1.000	0.000	0.000	0.383	0.667
$X_5$	0.714	0.800	1.000	0.000	0.333
Standard series ( $X_0$ )	1	1	1	1	1

**3.6 Determination of Grey relational coefficient for each evaluation factor**

1. Calculate the maximum and minimum difference by:

$$\Delta_{\max} = \max_{j? i} \max_{j? k} \|X_i(k) - X_j(K)\|,$$

the resulting maximum difference is one;

$$\Delta_{\min} = \min_{j? i} \min_{j? k} \|X_i(k) - X_j(K)\|,$$

the resulting minimum difference is zero.

2. Calculate the Grey relational coefficient by:

$$\gamma_{0i}(k) = \frac{\Delta_{\min} + \Delta_{\max}}{\Delta_{0i}(k) + \Delta_{\max}}.$$

By substituting the value of maximum and minimum difference into above equations, the Grey relational coefficient for each candidate vendor is shown in Table 5.

Table 5: The Grey relational coefficient

Item Comparative series	Quality	Price	Delivery date	Quantity	Services
$X_1$	0.667	0.555	0.714	1.000	0.500
$X_2$	0.500	0.714	0.555	0.833	0.750
$X_3$	0.667	1.000	0.714	1.000	1.000
$X_4$	1.000	0.500	0.500	0.618	0.750
$X_5$	0.777	0.833	1.000	0.500	0.560

### 3.7. Determination of the relational grade for each candidate vendor

Using the corresponding weighting value for each evaluation factor (see Table 3), we can calculate the relational grade of each candidate vendor by:

$$\Gamma_{0i} = \gamma(x_0, x_i) = \sum_{k=1}^n \beta_k \gamma(x_0(k), x_i(k)),$$

where  $\beta_k$  is the corresponding weighting value for each evaluation factor. Substituting the relational coefficient from Table 5 into above equation 6, the Grey relational coefficient can be obtained.

$$\Gamma_{01} = 0.602; \quad \Gamma_{02} = 0.601; \quad \Gamma_{03} = 0.791;$$

$$\Gamma_{04} = 0.618; \quad \Gamma_{05} = 0.659$$

This value of Grey relation is the overall performance that the enterprise requires.

### 3.8. Obtaining the ranking

Because of  $\Gamma_{03} > \Gamma_{05} > \Gamma_{04} > \Gamma_{01} > \Gamma_{02}$ , the ranking order for all candidate vendors is: (1) C; (2) E; (3) D; (4) A; (5) B. It is noted that the ranking order will change while we change the weighting value for each evaluation factor. In other words, the owner of an enterprise may select a suitable vendor based on his own requirements.

### 4. Conclusions

**In order to seek a proper vendor meeting the requirements of enterprise itself subjectively, it is important to develop an accurate evaluation method. Due to the advantages of Grey multiple attributes decision, this paper tries to propose an evaluation method to determine the overall performance for each candidate vendor. The optimum decision can then be made based on the overall performance. Moreover, from the equation derivation and a numeric example for vendor evaluation, this study obtains the following advantages:**

1. For new vendor evaluation, it is very convenient to perform overall measurement based on each enterprise's requirements. The overall performance can determine the order for selecting the suitable vendors.
2. The enterprise can choose its own appropriate goal and weighting value for each evaluation factor based on the characteristic demand of raw materials in order to select the most suitable vendors.
3. Based on the preceding discussion, it is judged that the proposed Grey multiple attributes decision method is very precise on the whole. It can overcome the ambiguity arising from the measured parameters of each attribute.



Therefore, the result of this study represents a further advance in the method of evaluating vendors.



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