# Applying Grey Relational Analysis to the Decathlon Evaluation Model

**Ching-Liang Chang, *Chih-Hung Tsai,** and **Lieh Chen**

Department of Industrial Engineering and Management
Ta-Hwa Institute of Technology
1 Ta-Hwa Road, Chung-Lin
Hsin-Chu, Taiwan, ROC
*E-mail: ietch@thit.edu.tw

***Abstract***

*Decathlon, track-and-field events consist of ten separate contests. Points are awarded for each event, and the overall score determines the winner. However, the class interval and its specified unit for scoring table seem to be unreasonable. To overcome this defect, Grey relational grade deduced by Grey theory [17] will be used to establish a complete and accurate evaluation model for determining who is the best all-around athlete among all contestants. This methodology not only will significantly reduce scoring disputes but also can help the attended team to select the best athletes. A numerical example for athletic scores ranking in a typical decathlon competition utilizing Grey relational analysis will also be made in this paper. This proposed method may provide the World Games or other sport federations with an improved score awarding method in order to determine who, among all athletes, is the best all-round athlete.*

***Keywords:*** *Grey theory, Grey relational analysis, Decathlon*

## 1. Introduction

The proposition of Grey theory occurring in the 1990 to 1999 time period resulted in the uses of Grey theory to each field, and the development is still going on. The major advantage of Grey theory is that it can handle both incomplete information and unclear problems very precisely. It serves as an analysis tool especially in cases when there is no enough data. It was recognized that the Grey relational analysis in Grey theory had been largely applied to project selection, prediction analysis, performance evaluation, and factor effect evaluation due to the Grey relational analysis software development. Recently, this technique has also applied to the field of sport and physical education. There are lots of good research results. For example, in the sport techniques, Hsu [11] used Grey relational analysis and determined the effects of striking and kicking action on overall scores. Wang [2] also took several factors affecting the release effect as multiple attributes to find how each factor influences the relation grade ranking for women javelin throw. The process of last force exertion matching up the action of left leg is studied in order to obtain highest scores. Yen [16] studied the interrelationship between defensive and offensive technique and team-win record through Grey relational analysis. Chen [13] had used Grey theory to make prediction for those soccer teams who can enter round of eight in 2000 World Cup based on the 1998 results of group preliminaries. Regarding score influence factor analysis for sport competition, Shen [7] applied Grey relational analysis to analyze and obtain the factors that influence Fu-En Lee's overall decathlon scores. He determined the Grey relation grade for overall score and ten individual event. Based on that, the ranking for those factors influence the overall score is obtained, and can be used as reference for future training. By using Grey relation analysis, Han [15] studied the effect of score variation in each station on overall score for top 10 athletes in Marathon Games in the period between 1985 and 1994 in order to understand which station most affect the overall score. For the body characteristics, Sun [12] took twelve items of characteristics from thirteen well-performance PRC athletes, and found six main body characteristics and its ranking order affecting the score by Grey relation analysis. Since the wide application of Grey relation analysis to sport competition and its advantage being inherent in the method, this paper aims to eliminate the unreasonableness regarding the class interval and point conversion, and provide a sound decathlon evaluation model for sport federation's reference.

## 2. Grey relational analysis

The Grey relational analysis uses information from the Grey system to dynamically compare each factor quantitatively. This approach is based on the level of similarity and variability among all factors to establish their relation. The relational analysis suggests how to make prediction and decision, and generate reports that make suggestions for the vender selection. This analytical model magnifies and clarifies the Grey relation among all factors. It also provides data to support quantification and comparison analysis [5]. In other words, the Grey relational analysis is a method to analyze the relational grade for discrete sequences. This is unlike the traditional statistics analysis handling the relation between variables. Some of its defects are: (1) it must have plenty of data; (2) data distribution must be typical; (3) a few factors are allowed and can be expressed functionally. But the Grey relational analysis requires less data and can analyze many factors that can overcome the disadvantages of statistics method. The Grey theory and method are described in the following:

### 2.1. Influence space, measurement space, and Grey relational space

Let P(X) represent the factor set of a specific topics, Q is the influence relation, then {P(X); Q} is influence space. It must have the following properties [6]:

1. Existence of key factors: for example, the key factors of basketball player are height, weight, and rebound.
2. Numbers of factors are limited and countable: for example each of the height, weight, and rebound are countable.
3. Factor independability: each factor must be independent.
4. Factor expandability: For example, besides the height, weight, and

rebound, the free throw attempt can be added as a factor.

The series formed by P(X) is:

$$x_i^{(0)}(k)=(x_1^{(0)}(1), ?, ??, x_i^{(0)}(k)) ? X;$$

where $i = 0,\cdots,m. \quad k = 1,\cdots,n. ? N$

If the following conditions are satisfied:

1. Nondimension: the numeric value for all factors must be nondimension.
2. Scaling: the factor value for various series must be at the same level.
3. Polarization: if the factor value in the series is described as the same direction, the series is comparable. Then the measurement space is expressed as {P(X); $x_i*(k)$}, the Grey relational space formed by the satisfaction of both factor space and comparability is termed by {P(X); $\Gamma$}.

## 2.2. Generation of Grey relation

Under the principle of series comparability, to achieve the purpose of Grey relational analysis, we must perform data processing. This processing is called generation of Grey relation or standard processing. The expected goal for each factor is determined by Wu [8, 9] based on the principles of data processing. They are described in the following:

1. If the expectancy is larger-the-better (e.g., the benefit), then it can be expressed by

$$x_{ij} = \frac{X_{ij} - (X_{ij})_{\min}}{(X_{ij})_{\max} - (X_{ij})_{\min}} \quad (1)$$

2. If the expectancy is smaller-the-better (e.g., the cost and defects), then it can be expressed by

$$x_{ij} = \frac{(X_{ij})_{\max} - X_{ij}}{(X_{ij})_{\max} - (X_{ij})_{\min}} \quad (2)$$

3. If the expectancy is nominal-the-best (e.g., the age), and when the targeted value is $X_o : (X_{ij})_{\max} ? X_o ? (X_{ij})_{\min}$, then it can be expressed by

$$x_{ij} = \frac{\left|X_{ij} - X_O\right|}{(X_{ij})_{\max} - X_O} \quad (3)$$

## 2.3. The Grey relational grade

The measurement formula for quantification in Grey relational space is called the Grey relational grade. When we are determining Grey relation and taking only one series, $x_0(x)$, as a referenced series, it is called the grade of local Grey relation. If anyone of the series, $x_i(x)$, is referenced series, it is called the grade of global Grey relation. Additionally, the Grey relational coefficient must first be determined before we obtain the Grey relational grade.

In the Grey relational space, {P(X); $\Gamma$}, there is a series

$$x_i = (x_i(1), x_i(2), ?, ??, x_i(k)) ? X$$

where $i = 0,\cdots,m. \quad k = 1,\cdots,n. ? N$

If the grade of local Grey relation is brought to define the Grey relational coefficient, $\gamma(x_i(k), x_j(k))$, it can be expressed as following:

$$\gamma\ (x_i(k), x_j(k)) = \frac{\Delta_{\min.} + \Delta_{\max.}}{\Delta_{0i}(k) + \Delta_{\max.}}$$
(4)

where $i = 0,\cdots,m.\quad k = 1,\cdots,n.\quad j ? i$;

- $x_0$ is a referenced series, $x_i$ is a specific comparative series;
- $\Delta_{0i} = \|x_0(k) - x_i(k)\|$: representing the $k$'s absolute value of the difference of $x_0$ and $x_i$;
- $\Delta_{min.} = \underset{\forall j ? i\ \forall k}{min.\ min.} \|x_0(k) - x_j(k)\|$;
- $\Delta_{max.} = \underset{\forall j ? i\ \forall k}{max.\ max.} \|x_0(k) - x_j(k)\|$.

After obtaining the Grey relational coefficient, we normally take the average of the Grey relational coefficient as the Grey relational grade:

$$\Gamma = \gamma\ (x_i, x_j) = \frac{1}{n} \overset{n}{\underset{k=1}{?}}\ \gamma\ (x_i(k), x_j(k))$$
(5)

However, since in real application the effect of each factor on the system is not exactly same, Eq. (5) can be modified as:

$$\Gamma = \gamma\ (x_i, x_j) = \overset{n}{\underset{k=1}{?}}\ \beta_k\ \gamma\ (x_i(k), x_j(k))$$
(6)

where $\beta_k$ represents the normalized weighting value of a factor and $\overset{n}{\underset{k=1}{?}}\ \beta_k = 1$ when equating both Eq (5) and (6).

## 2.4. The Grey relational series

The Grey relational grade represents the correlation between two series. It is not important in a decision-making. Rather, the ranking order of the relational grade is the most important information. Therefore, *m's* comparative series with its corresponding Grey relational grade is rearranged according to the order of the their magnitudes. A Grey relational series is defined as following:

In the Grey relational space, {P(X); Γ}, referenced series, $x_0$ , and comparative series, $x_i$ and $x_j$:

$x_0 = (x_0(k))$, $k = 1, \ldots , n.$
$x_i = (x_i(k))$, $k = 1, \ldots , n;\ i ?$ I
$x_j = (x_j(k))$, $k = 1, \ldots , n;\ j ?$ I

If $\gamma(x_0, x_i)\ ?\ \gamma(x_0, x_j)$, the situation indicating the relational grade of $x_i$ vs. $x_0$ is greater than that of $x_j$ vs. $x_0$ , or represented by $\Gamma_{0i} > \Gamma_{0j}$. This is the relational series for $x_i$ and $x_j$ [9].

## 3. Traditional score evaluation model versus Grey relational model

Decathlon, track-and-field events consisting of 10 separate points contests held on two consecutive days. Points are awarded for each event [3], and the overall score determines the winner. The athlete receives predetermined points for reaching certain distances, heights, and times in the events. Then cumulative score from each event score determines the places of the athlete. The international track-and-field rule for Decathlon [4] can use the following utility function:

$$TS = \overset{n}{\underset{i=1}{?}}\ S_i$$
(7)

Where *TS* is the total score, $S_i$ is the individual event score, and *i* range from 1 to 10.

Regarding the places determined by above method, there are some scoring disputes, which are (1) According to scoring table, each event reaching certain distance (meter), height (meter), and time (second) are converted into event scores. Total points are then obtained by summing up each individual event points. But whether this conversion is reasonable is still questionable. (2) The measurement unit is same for some events, but the class interval in conversion table are not the same. For example, the class interval for Pole Vault in Table 1 showing 2 points or 3 points difference for every 0.01 meter seems to be unreasonable, but 1 point difference for every 0.06 meter in Javelin Throw. (3) Each event score represents the athlete possessing some degree of characteristic such as muscular strength, muscular endurance, cardiovascular endurance, etc. Simply converting the measurement into points and summing up seem to be questionable. (4) There is no way to decide the winner if there is a tie-score.

On the other hand, using Grey relation analysis to perform multiple attributes overall evaluation, the measurement unit is not necessarily the same. It can easily determine the overall index without consulting utility function. Moreover, the decathlon athlete selection possesses those above-mentioned properties. Hence, it is appropriate to adopt Grey relational analysis to decide the ranking for decathlon athlete.

## 4. Illustration of numerical results

For showing the significance of Grey relational analysis in sport score evaluation and resolving the problems of scoring dispute, the method for decathlon ranking using Grey relational analysis is discussed herein. We assume there are five contestants attending decathlon competition. The score is shown in Table 2. By utilizing traditional method, the results in Table 3 showing the ranking order are A, D, B, C and E.

Table 1: Typical score table for Pole Vault and Javelin Throw

| Pole Vault | | Javelin Throw | |
|---|---|---|---|
| Meters | Points | Meters | Points |
| 3.45 | 469 | 101.26 | 1,375 |
| 3.44 | 467 | 101.20 | 1,374 |
| 3.43 | 464 | 101.14 | 1,373 |
| 3.42 | 462 | 101.08 | 1,372 |
| 3.41 | 459 | 101.02 | 1,371 |
| 3.40 | 457 | 100.96 | 1,370 |

Table 2: Statistical data for the decathlon competition

| **Event** | 100 M (Secs) | Long Jump | Shot Put | High Jump | 400 M (Secs) | 110 M High | Discus Throw | Pole Vault | Javelin Throw | 1500 M (Secs) |
|---|---|---|---|---|---|---|---|---|---|---|

| Contestant | | (M) | (M) | (M) | | Hurdles (Secs) | (M) | (M) | (M) | |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 11.13 | 7.34 | 14.50 | 2.25 | 48.45 | 14.56 | 43.67 | 5.70 | 60.50 | 256.65 |
| B | 11.10 | 6.95 | 13.75 | 2.27 | 47.20 | 14.18 | 45.50 | 5.12 | 55.25 | 265.55 |
| C | 11.65 | 6.50 | 12.80 | 2.08 | 46.90 | 14.05 | 39.55 | 4.45 | 60.15 | 290.15 |
| D | 11.25 | 7.43 | 14.25 | 2.27 | 48.90 | 15.13 | 49.28 | 4.70 | 61.32 | 240.95 |
| E | 11.00 | 7.23 | 13.15 | 2.03 | 49.73 | 14.96 | 38.66 | 4.50 | 52.82 | 288.57 |

Table 3: Event score obtained by traditional method

| Event / Contestant | 100 M | Long Jump | Shot Put | High Jump | 400 M | 110 M High Hurdles | Discus Throw | Pole Vault | Javelin Throw | 1500 M | Total scores | Rank |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 832 | 896 | 759 | 1041 | 887 | 903 | 740 | 1090 | 746 | 839 | 8,733 | 1 |
| B | 838 | 802 | 713 | 1061 | 948 | 951 | 777 | 947 | 667 | 778 | 8,482 | 3 |
| C | 721 | 697 | 655 | 878 | 963 | 968 | 655 | 746 | 740 | 619 | 7,642 | 4 |
| D | 806 | 918 | 744 | 1061 | 866 | 834 | 855 | 819 | 758 | 885 | 8,546 | 2 |
| E | 861 | 869 | 676 | 831 | 829 | 854 | 638 | 760 | 630 | 631 | 7,599 | 5 |

Moreover, if five contestants attending decathlon competition, the following procedure indicates how the top of three can be determined.

### 4.1. Implementation with evaluation matrix

Generate an evaluation matrix by arranging game event as attribute column, and contestants as comparative sequence. In the analysis, the record for the five contestants is used to form evaluation matrix as shown in Table 2.

### 4.2. Data rationalizing

Expected goal can be rationalized according to each attribute. A group of assumptions are made for the following:

1. 100 meters, 400 meters, 1500 meters, and 110 meters high hurdles: In this category, it is obviously that the expectancy is shorter-the-better for the time, which can be determined by:

$$x_{ij} = \frac{(X_{ij})_{\max} - X_{ij}}{(X_{ij})_{\max} - (X_{ij})_{\min}}$$

In which $X_{ij}$ represents the score both at attribute $i$ and comparative series $j$.

2. Long Jump, Shot Put, and High Jump: The expectancy in this category is longer-the-better for the distance, which can be determined by:

$$x_{ij} = \frac{X_{ij} - (X_{ij})_{\min}}{(X_{ij})_{\max} - (X_{ij})_{\min}}$$

Based on the expectancy of each individual event, the scoring points for each attribute are normalized to obtain the matrix table of comparative series as shown in Table 4.

### 4.3. Establishing standard series

In accordance with our expected goal for each individual contest event, an ideal standard series ($X_0 = 1$) is established in the last line in Table 4.

Table 4: Data rationalizing

| Event / Comparative Series | 100 M | Long Jump | Shot Put | High Jump | 400 M | 110 M High Hurdles | Discus Throw | Pole Vault | Javelin Throw | 1500 M |
|---|---|---|---|---|---|---|---|---|---|---|
| $X_1$ | 0.800 | 0.903 | 1.000 | 0.912 | 0.452 | 0.528 | 0.472 | 1.000 | 0.904 | 0.681 |
| $X_2$ | 0.846 | 0.484 | 0.559 | 1.000 | 0.894 | 0.880 | 0.644 | 0.536 | 0.286 | 0.500 |
| $X_3$ | 0.000 | 0.000 | 0.000 | 0.208 | 1.000 | 1.000 | 0.084 | 0.000 | 0.862 | 0.000 |
| $X_4$ | 0.615 | 1.000 | 0.853 | 1.000 | 0.293 | 0.000 | 1.000 | 0.200 | 1.000 | 1.000 |
| $X_5$ | 1.000 | 0.785 | 0.258 | 0.000 | 0.000 | 0.157 | 0.000 | 0.040 | 0.000 | 0.032 |
| Standard series ($X_0$) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

### 4.4. Determination of Grey relational coefficient for each contestant

1. Calculate the maximum and minimum difference by:

$$\Delta_{\max} = \underset{\forall j ?\, i}{\max.} \ \underset{\forall j ?\, k}{\max.} \left\| X_i(k) - X_j(K) \right\|$$, the resulting maximum difference is one;

$$\Delta_{\min} = \underset{\forall j ?\, i}{\min.} \ \underset{\forall j ?\, k}{\min.} \left\| X_i(k) - X_j(K) \right\|,$$ the resulting minimum difference is zero.

2. Calculate the Grey relational coefficient by:

$$\gamma_{0i}(k) = \frac{\Delta_{\min} + \Delta_{\max}}{\Delta_{0i}(k) + \Delta_{\max}}.$$

By substituting the value of maximum and minimum difference into above equations, the Grey relational coefficient for each contestant (individual contest event) is shown in Table 5.

Table 5: The Grey relational coefficient

| Event / Comparative | 100 M | Long Jump | Shot Put | High Jump | 400 M | 110 M High Hurdles | Discus Throw | Pole Vault | Javelin Throw | 1500 M |
|---|---|---|---|---|---|---|---|---|---|---|

| series | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $X_1$ | 0.833 | 0.911 | 1.000 | 0.919 | 0.646 | 0.679 | 0.654 | 1.000 | 0.912 | 0.758 |
| $X_2$ | 0.866 | 0.659 | 0.693 | 1.000 | 0.904 | 0.893 | 0.737 | 0.683 | 0.583 | 0.667 |
| $X_3$ | 0.500 | 0.500 | 0.500 | 0.558 | 1.000 | 1.000 | 0.522 | 0.500 | 0.879 | 0.500 |
| $X_4$ | 0.722 | 1.000 | 0.871 | 1.000 | 0.585 | 0.500 | 1.000 | 0.556 | 1.000 | 1.000 |
| $X_5$ | 1.000 | 0.823 | 0.574 | 0.500 | 0.500 | 0.543 | 0.500 | 0.510 | 0.500 | 0.508 |

### 4.5. Determination of the relational grade for each contestant

The relation grade for each comparative series is determined by averaging the Grey relation coefficient of each individual contest event. The Grey relation grade can be expressed by:

$$\Gamma_{0i} = \frac{1}{n} \overset{n}{\underset{k=1}{?}} \gamma(x_0(k), x_i(k)),$$

where $n = 10$

Substituting the coefficient of Grey relation into above equation, we can get each contestant's Grey relation grade, which are

$\Gamma_{01}= 0.832$; $\Gamma_{02}= 0.769$; $\Gamma_{03}= 0.646$;

$\Gamma_{04}= 0.823$; $\Gamma_{05}= 0.596$.

This Grey relation grade is the overall performance for the decathlon.

### 4.6. Obtaining the ranking

Since we get $\Gamma_{01} > \Gamma_{04} > \Gamma_{02} > \Gamma_{03} > \Gamma_{05}$, the ranking order for these five contestants is A, D, B, C and E. Although Grey relational analysis does agree well with the traditional method, it possesses an overwhelming advantage to solve the problems that traditional method could not overcome such as tie score dispute.

## 5. Conclusions

Based on our study in this paper, the Grey relational analysis can be applied in analyzing sport technology, selection of coach, and evaluation of overall performance for decathlon. Through quantitative analysis of Grey relation, it provides more accurate and subjective data. The method is formulated in tabulated form so that it can be conveniently used for other sport evaluations. In addition, there is no need to make score conversion or consulting utility function. We believe this method can provide a sound analytical approach to the decathlon evaluation model.

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