Improvement of Electromagnetic Wave Propagation Using Novel Finite Element Methodology

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Abstract - **This research presented a new numerical analysis approach to improve simulation software by applying the finite element strategy for electromagnetic problem. We present the analyzed patterns of electromagnetic wave and propagation in various medium. This will be able to decrease education and research related budgets. Nowadays, the commercial software is costly and cannot adjustable the source code so that is not sufficient to simulation in research works. The simulation program is synthesized and designed to solve the complex boundary condition for electromagnetic wave propagation. The simulation reflects the effective numerical analysis approach presented by using MATLAB. It can be applied in engineering education as well as can be used to solve complicated electromagnetic issues.**

Keywords - **Finite Element Method, Simulation Program, Electromagnetic Wave Propagation**

I. INTRODUCTION

Recently, the technology development is essential in communication system. Usually the high frequency systems in communication technology are developed in both education and research. There are several computational methods used to solve electromagnetic wave problems such as the Method of Moment (MoM) [1], the Finite Difference Time Domain (FDTD) [2-5], Finite Difference Method (FDM) [6] and the Finite Element Method (FEM) [7-8]. Therefore, the development of various numerical methods is necessary for construction of an efficient electromagnetic simulation tool [9]. In basic MoM method, its recorded antecedents would a long way more seasoned over this, for this the event going back to the nineteenth century and the vibrational techniques initially portrayed by Lord Rayleigh. It is also widely utilized as a part of structure mechanics nowadays, and additionally to computational fluid dynamics, computational thermodynamics, the numerical arrangement of Schrödinger's equation, field issues in general, and of course, in electromagnetic. With the foundation we bring presently obtained with the FDTD also MoM, on looker will distinguish numerous characteristics in a similar manner as both of these techniques in the treatment to take after in reality, they will likely not be astonished to realize every one of the three that can be planned inside of a weighted remaining setting up, how to do this for the FDTD is not instantly obvious. In a similar manner as that of the MoM, the main concept is to displace some obscure capacity on a domain by a gathering of elements, with referred to shape however obscure plentiful. Different from the individuals crucial FDTD, the place the close estimation of the electric and magnetic fields will be ceaselessly done with admiration to a rectangular, staggered grid, the FEM permits greatly all geometrical segments once a chance to be used also (usually) main utilization one grid. The most generally utilized elements are known as simplifies this just means line elements in 1D, triangular in 2D, and tetrahedral in 3D. None the less, rectangular, kaleidoscopic and even curvilinear elements likewise find across the board application. Subsequent to the enhanced geometrical modeling made conceivable particularly by triangular or tetrahedral meshes is one of the real characteristics separating the FEM from the FDTD. Our investigation of the FEM will be to a great extent confined to these elements. Intrigued peruses might discover treatments of other element shapes in the references. While it is similar to the FDTD, and partially like the MOM, the FEM depends on a local description of the field quantities, determined from the differential equation portrayal of the Maxwell's equations, and does not consequently join the Somerfield radiation condition. On practice, this intends a few structure of mesh termination scheme is required.

This paper presents the method of using the Finite Element Method (FEM) to solve the electromagnetic wave propagation problems in high frequency circuits. The analysis model based on Finite Element Method was implemented by using the Graphic User Interface (GUI) function of MATLAB software. Users are able to define the dimensions of the conducting plate and select the shape of patch conductor also cavity dimension and number of mode for analysis of the electromagnetic wave in the spectral domain in various mediums. Therefore, the development of numerical methods is important for an efficient electromagnetic simulation tool.

II. THEORY

A. Wave Propagation

Computation for the electromagnetic wave propagation includes amplitude, direction of the incident, reflected and transmitted waves that propagate in both conductor and medium. The waves can be calculated in the real domain in conducting plan and spectrum domain scattering in waveguide concern about frequency of wave in TE and TM mode, the conductivity shown in Fig. 1.

For Finite Element Method, the conducting plan can be analyzed in many shapes of conductor, this is the advantage of this method. Shape of conductor in this study can be replaced by any shape in Fig. 2.

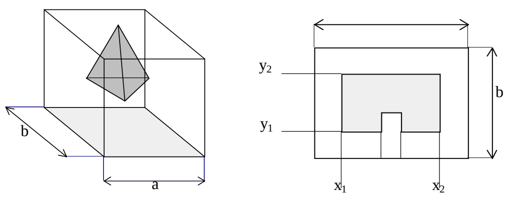


Fig. 1 The Conductor in a Box

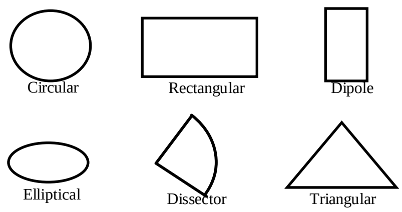


Fig. 2 Shape of Conductor

Electric fields for transverse electric (TE) mode by following equation is:

$$E_x^{TE} = \frac{n}{b\sqrt{\frac{n^2}{b^2}+\frac{m^2}{a^2}}}\sqrt{\frac{2\tau_{mn}}{ab}}\,cos\left(\frac{m\pi x}{a}\right)sin\left(\frac{n\pi y}{b}\right)$$
$$E_y^{TE} = \frac{-m}{a\sqrt{\frac{n^2}{b^2}+\frac{m^2}{a^2}}}\sqrt{\frac{2\tau_{mn}}{ab}}\,sin\left(\frac{m\pi x}{a}\right)cos\left(\frac{n\pi y}{b}\right) \quad (1)$$

Electric fields for transverse magnetic (TM) mode by following equation is:

$$E_x^{TM} = \frac{n}{b\sqrt{\frac{n^2}{b^2}+\frac{m^2}{a^2}}}\sqrt{\frac{2\tau_{mn}}{ab}}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$
$$E_y^{TM} = \frac{-m}{a\sqrt{\frac{n^2}{b^2}+\frac{m^2}{a^2}}}\sqrt{\frac{2\tau_{mn}}{ab}}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) \quad (2)$$

While,

$\tau_{mn} = 2$ If m and n are not equal to zero.

$\tau_{mn} = 1$ If m and n are equal to zero.

The boundary value problem of conducting plan, the conductor in medium, can be computed and analyzed by using boundary problem of exact solution for half contact to the space and half contact to the conducting plan. In the operation, it can be analyzed by waveguide equation, as shown in Fig. 3. The same method of computation and analysis by using waveguide equation is also applied to the conducting cavity.

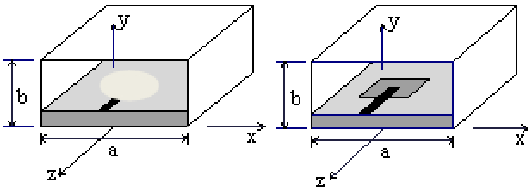


Fig. 3 The Conductor Placed in Conducting Cavity

The Eigen values; γ^2 and cutoff wavenumbers; γ, can be calculations were carried out Eigen value shown in (3)

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{a/2}\right)^2} \qquad (3)$$

B. Finite Element Method (FEM)

The Finite Element Method can solve the problem solution in many dimensions, such as the element shown in Fig. 4. The Finite Element Method has operating process following.

1. **Pre-Processing Step**

The preprocessing step requires the automatic mesh generator. It divides the region area under learn into a set of elements, mostly triangles that are fit in a generic two dimensional shape. The mesh generator creates following information about the mesh, as shown in Fig. 5.

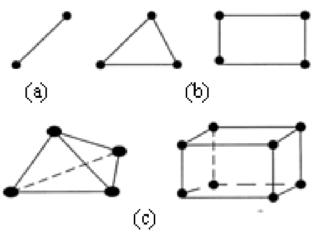


Fig. 4 (a) One-Dimension,
(b) Two-Dimension, and (c) Three-Dimension

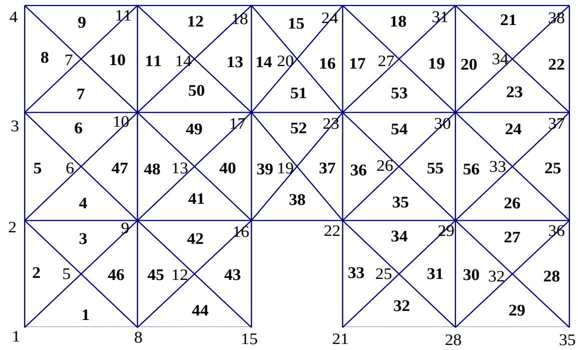


Fig. 5 Mesh and Node of Interesting Region

The mesh data must be computed by the LAPLACE to solve the problem properly. These are contained in Table I, II, and III. Table I displays, lists of the node coordinates and Table II displays, lists of the nodes that are driblet boundary conditions, and Table III displays, lists of the entire element and described nodes.

2. **Building the Matrix**

The unknown function is approximated each element used the polynomial expression.

$$Q(x,y) = a + bx + cy = \begin{bmatrix} 1 & x & y \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \qquad (4)$$

The approximated Cartesian coordinate use function potential assumes the triangle node value.

TABLE I
NODE COORDINATE

Node No.	X	Y	Node No.	X	Y
1	0.00	0.00	20	0.50	0.84
2	0.00	0.33	21	0.60	0.00
3	0.00	0.67	22	0.60	0.33
4	0.00	1.00	23	0.60	0.67
5	0.10	0.17	24	0.60	1.00
6	0.10	0.50	25	0.70	0.17
7	0.10	0.84	26	0.70	0.50
8	0.20	0.00	27	0.70	0.84
9	0.20	0.33	28	0.80	0.00
10	0.20	0.67	29	0.80	0.33
11	0.20	1.00	30	0.80	0.67
12	0.30	0.17	31	0.80	1.00
13	0.30	0.50	32	0.90	0.17
14	0.30	0.84	33	0.90	0.50
15	0.40	0.00	34	0.90	0.84
16	0.40	0.33	35	1.00	0.00
17	0.40	0.67	36	1.00	0.33
18	0.40	1.00	37	1.00	0.67
19	0.50	0.50	38	1.00	1.00

$$\begin{cases} Q_1(x_1, y_1) = a + bx_1 + cy_1 \\ Q_2(x_2, y_2) = a + bx_2 + cy_2 \\ Q_3(x_3, y_3) = a + bx_3 + cy_3 \end{cases} \quad (5)$$

The equation in matrix form can be rewritten as:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (6)$$

While, A is the area of element.

$$A = \frac{1}{2} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} = (x_1 y_2 - x_2 y_1) + (x_3 y_1 - x_1 y_3) + (x_2 y_3 - x_3 y_2) \quad (7)$$

Introducing equation (7) into (4) yields,

$$Q(x,y) = \frac{1}{2A} [1 \quad x \quad y] \times \begin{bmatrix} x_2 y_3 - x_3 y_2 & x_3 y_1 - x_1 y_3 & x_1 y_2 - x_2 y_1 \\ y_2 - y_3 & y_3 - y_1 & y_1 - y_2 \\ x_3 - x_2 & x_1 - x_3 & x_2 - x_1 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} \quad (8)$$

As shown in Fig. 6, the two properties are presented as,

$$\alpha_i = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases} \quad (9)$$

Where;

$$\alpha_1 = \frac{1}{2A} [(x_2 y_3 - x_3 y_2) + (y_2 - y_3) x + (x_3 - x_2) y],$$

$$\alpha_2 = \frac{1}{2A} [(x_3 y_1 - x_1 y_3) + (y_3 - y_1) x + (x_1 - x_3) y],$$

$$\alpha_3 = \frac{1}{2A} [(x_1 y_2 - x_2 y_1) + (y_1 - y_2) x + (x_2 - x_1) y]$$

The functional element can be expressed as in (10).

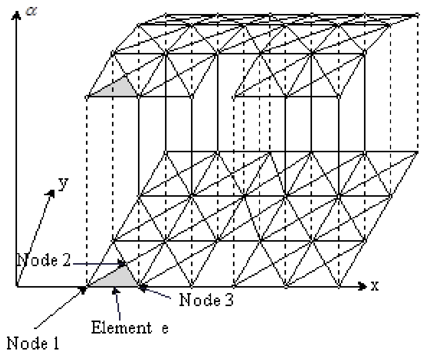
$$P^{(e)} = \frac{1}{2} \in^{(e)} \sum_{i=1}^{3} \sum_{j=1}^{3} Q_i Q_j \int_{\Delta} (e) \nabla_t \alpha_i \cdot \nabla_t \alpha_j \, dR \quad (10)$$

It can be rewritten in equation of matrix form.

$$P^{(e)} = \frac{1}{2} \in^{(e)} [Q]^t [R^{(e)}][Q] \quad (11)$$

TABLE II
NODE COORDINATE

Node No.	Lb.	Prescribed Potential	Node No.	Lb.	Prescribed Potential
1	1	0.00	22	2	1.00
2	1	0.00	24	1	0.00
3	1	0.00	28	1	0.00
4	1	0.00	31	1	0.00
11	1	0.00	35	1	0.00
15	2	1.00	36	1	0.00
16	2	1.00	37	1	0.00
18	1	0.00	38	1	0.00
21	2	1.00			



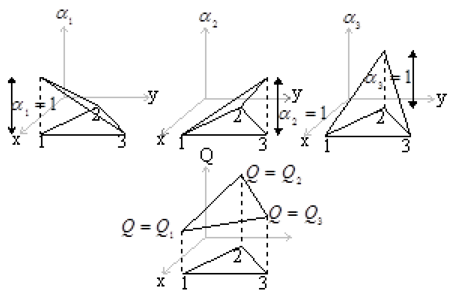


Fig. 6 Shape Functions and Linear Approximation for a Triangular Element

$$\text{With} [Q] = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}, \; [R^{(e)}] = \begin{bmatrix} R_{11}^{(e)} & R_{12}^{(e)} & R_{13}^{(e)} \\ R_{21}^{(e)} & R_{22}^{(e)} & R_{23}^{(e)} \\ R_{31}^{(e)} & R_{32}^{(e)} & R_{33}^{(e)} \end{bmatrix} \quad (12)$$

Where;

$$R_{11}^{(e)} = \frac{1}{4A} \left[(y_2 - y_3)^2 + (x_3 - x_2)^2 \right],$$

$$R_{12}^{(e)} = \frac{1}{4A} [(y_2 - y_3)(y_3 - y_1) + (x_3 - x_2)(x_1 - x_3)],$$

$$R_{13}^{(e)} = \frac{1}{4A} [(y_2 - y_3)(y_1 - y_2) + (x_3 - x_2)(x_2 - x_1)],$$

$$R_{22}^{(e)} = \frac{1}{4A}\left[(y_3 - y_1)^2 + (x_1 - x_3)^2\right],$$

$$R_{33}^{(e)} = \frac{1}{4A}\left[(y_1 - y_2)^2 + (x_2 - x_1)^2\right],$$

$$R_{23}^{(e)} = \frac{1}{4A}\left[(y_3 - y_1)(y_1 - y_2) + (x_1 - x_3)(x_2 - x_1)\right],$$

Also,

$$R_{21}^{(e)} = R_{12}^{(e)}, \quad R_{31}^{(e)} = R_{13}^{(e)}, \quad R_{32}^{(e)} = R_{23}^{(e)}$$

TABLE III
CONNECTION BETWEEN GLOBAL AND LOCAL NUMBERING SCHEMES

El. No.	Lb.	Local Node Number 1	2	3	El. No.	Lb.	Local Node Number 1	2	3	El. No.	Lb.	Local Node Number 1	2	3
1	2	1	5	8	20	1	30	31	34	39	1	16	17	19
2	2	1	2	5	21	1	31	34	38	40	1	13	16	17
3	2	2	5	9	22	1	34	37	38	41	1	9	13	16
4	1	2	6	9	23	1	30	34	37	42	2	9	12	16
5	1	2	3	6	24	1	30	33	37	43	2	12	15	16
6	1	3	6	10	25	1	33	36	37	44	2	8	12	15
7	1	3	7	10	26	1	29	33	36	45	2	8	9	12
8	1	3	4	7	27	2	29	32	36	46	2	5	8	9
9	1	4	7	11	28	2	32	35	36	47	1	6	9	10
10	1	7	10	11	29	2	28	32	35	48	1	9	10	13
11	1	10	11	14	30	2	28	29	32	49	1	10	13	17
12	1	11	14	18	31	2	25	28	29	50	1	10	14	17
13	1	14	17	18	32	2	21	25	28	51	1	17	20	23
14	1	17	18	20	33	2	21	22	25	52	1	17	19	23
15	1	18	20	24	34	2	22	25	29	53	1	23	27	30
16	1	20	23	24	35	1	22	26	29	54	1	23	26	30
17	1	23	24	27	36	1	22	23	26	55	1	26	29	30
18	1	24	27	31	37	1	19	22	23	56	1	29	30	33
19	1	27	30	31	38	1	16	19	22					

3. Assembling the Global Matrices

It contains the unknown problem which can be written in equation as,

$$\begin{aligned} P &= P^{(1)} + P^{(2)} \\ P^{(1)} &= \frac{1}{2} \in^{(1)} \left[Q_g^{(1)}\right]^t \left[R^{(1)}\right]\left[Q_g^{(1)}\right] \\ P^{(2)} &= \frac{1}{2} \in^{(2)} \left[Q_g^{(2)}\right]^t \left[R^{(2)}\right]\left[Q_g^{(2)}\right] \\ P &= \frac{1}{2}\left[Q\right]^t \left[R\right]\left[Q\right] \end{aligned} \tag{13}$$

Where $[R]$ are the global 4x4 matrices,

$$[R] = \in^{(1)} \begin{bmatrix} R_{11}^{(1)} & R_{12}^{(1)} & R_{13}^{(1)} & 0 \\ R_{21}^{(1)} & R_{22}^{(1)} & R_{23}^{(1)} & 0 \\ R_{31}^{(1)} & R_{32}^{(1)} & R_{33}^{(1)} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \in^{(2)} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & R_{11}^{(2)} & R_{13}^{(2)} & R_{12}^{(2)} \\ 0 & R_{31}^{(2)} & R_{33}^{(2)} & R_{32}^{(2)} \\ 0 & R_{21}^{(2)} & R_{23}^{(2)} & R_{22}^{(2)} \end{bmatrix}$$

$$[R] = \begin{bmatrix} R_{11}^{(1)} & R_{12}^{(1)} & 0 & R_{13}^{(1)} \\ R_{21}^{(1)} & R_{22}^{(1)} + R_{11}^{(2)} & R_{12}^{(2)} & R_{23}^{(1)} + R_{12}^{(2)} \\ 0 & R_{21}^{(2)} & R_{22}^{(2)} & R_{23}^{(2)} \\ R_{31}^{(1)} & R_{32}^{(1)} + R_{31}^{(2)} & R_{32}^{(2)} & R_{33}^{(1)} + R_{33}^{(2)} \end{bmatrix} \tag{14}$$

The finite element mesh consists of three elements, as shown in Fig. 7. The global coefficient matrix can find values, as shown in (15).

$$[R] = \begin{bmatrix} R_{11}^{(1)} & R_{12}^{(1)} & 0 & R_{13}^{(1)} & 0 \\ R_{21}^{(1)} & R_{22}^{(1)}+R_{11}^{(2)}+R_{11}^{(3)} & R_{12}^{(3)} & R_{23}^{(1)}+R_{13}^{(2)} & R_{12}^{(3)}+R_{13}^{(3)} \\ 0 & R_{21}^{(3)} & R_{22}^{(3)} & 0 & R_{23}^{(3)} \\ R_{31}^{(1)} & R_{32}^{(1)}+R_{31}^{(2)} & 0 & R_{33}^{(1)}+R_{33}^{(2)} & R_{32}^{(2)} \\ 0 & R_{21}^{(2)}+R_{31}^{(3)} & R_{32}^{(3)} & R_{23}^{(2)} & R_{22}^{(2)}+R_{33}^{(3)} \end{bmatrix} \tag{15}$$

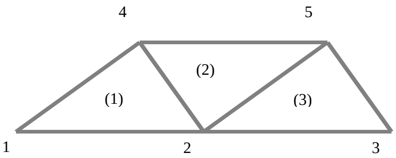


Fig. 7 Three-Triangle Meshes with Local and Global Node Numbering.

4. Minimizing the Function

We avoid unnecessarily complex node to the matrix of linear equation,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & R_{22} & R_{23} & 0 \\ 0 & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} Q_1 \\ -R_{11}\overline{Q}_1 - R_{14}\overline{Q}_4 \\ -R_{41}\overline{Q}_1 - R_{44}\overline{Q}_4 \\ \overline{Q}_4 \end{bmatrix} \quad (16)$$

5. Post-Processing Step

As mentioned above, the minimum value which can assume as $Q(x,y) = \overline{Q}(x,y)$ represents the stored energy $\overline{P}$ in the FEM computation, as shown in (18).

$$\overline{P} = \frac{1}{2}\left[\overline{Q}\right]^t [S]\left[\overline{Q}\right] \quad (17)$$

C. Electromagnetic Simulation Design

The electromagnetic simulation was created through a graphical user interface (GUI) function of MATLAB program, as shown in Fig. 8 and computing process of electromagnetic simulation using novel FEM method for analysis of a conducting patch placed in cavity was shown in Fig. 9 and Fig. 10.

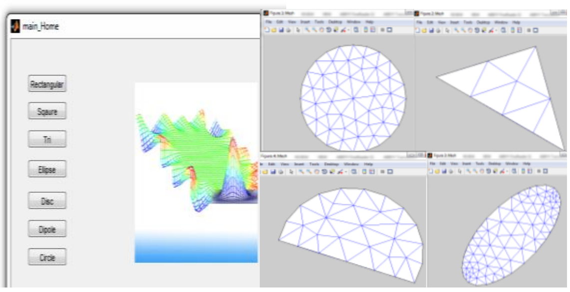


Fig. 8 Electromagnetic Simulation Program

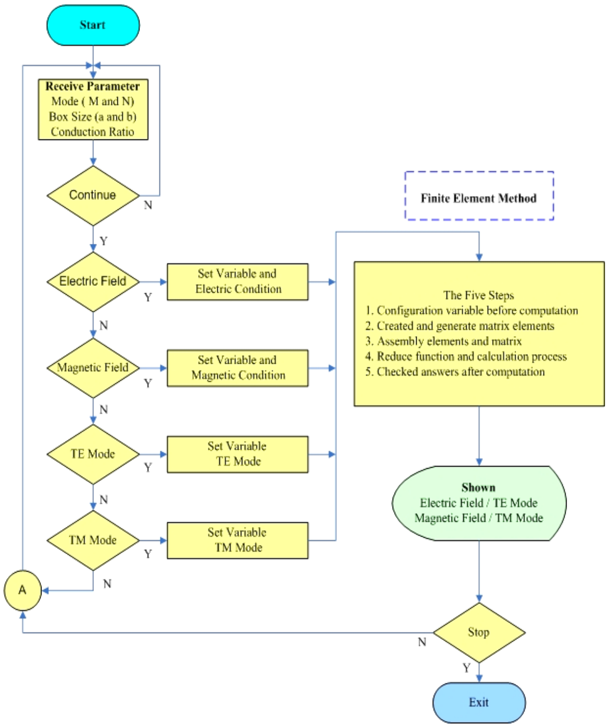


Fig. 9 Flowchart of Electromagnetic Simulation

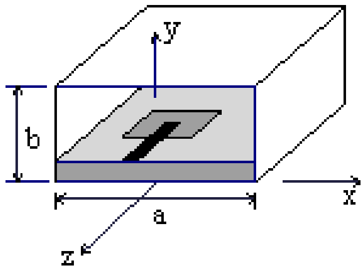


Fig. 10 The Conductor Placed in Conducting Cavity

III. RESULTS AND DISCUSSION

The developed electromagnetic simulation program can compute and show accuracy electric and magnetic field propagated on the conducting patch in gravity, as shown in Fig. 10 including graph pattern of TE and TM mode to be consistent to electromagnetic theory, as shown in Fig. 11 and 12 and the eigenvalues were shown in table IV and V. The advantage of this research can analyze various conducting shape in medium. Thus, the presentation of approached Finite Element Method can be applied to design and analyze high frequency circuits, furthermore telecommunication engineering.

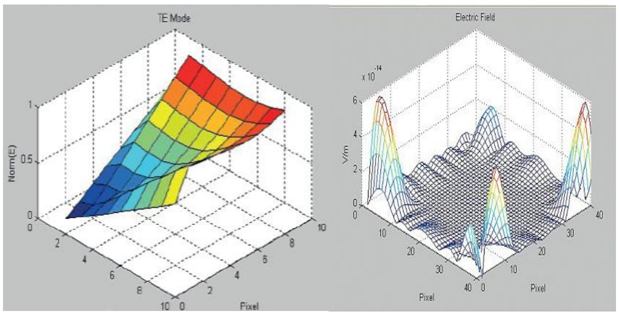


Fig. 11 Electric and Electric Field in TE Mode

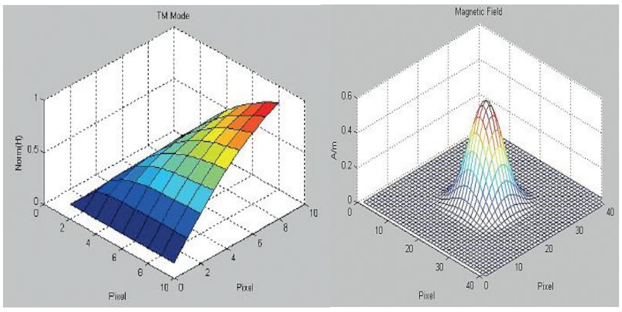


Fig. 12 Magnetic and Magnetic Field in TM Mode

TABLE IV
ANALYTICAL AND NUMERICAL
γa VALUES FOR TE MODE

Mode	Analytical Result	FEM Result
TE_{10}	3.14	3.14
TE_{20}	6.28	6.28
TE_{01}	6.28	6.28
TE_{11}	7.02	7.02
TE_{21}	8.88	8.88
TE_{30}	9.42	9.42

TABLE V
ANALYTICAL AND NUMERICAL
γa VALUES FOR TM MODE

Mode	Analytical Result	FEM Result
TM_{11}	7.02	7.02
TM_{21}	8.88	8.88
TM_{31}	11.33	11.33
TM_{12}	12.95	12.95
TM_{22}	14.05	14.04
TM_{32}	15.71	15.70

Table IV and V shown the results compared with analytical and Finnite Element Method (FEM) in cutoff frequency or eigenvalues of transverse electric (TE) and transverse magnetic (TM) fields. The result can be shown FEM perfermance consistent with conventional numerical method.

IV. CONCLUSIONS

In this paper, the simple electromagnetic field problems were presented in time and frequency domain and design of complex electromagnetic wave to solve more complicated solutions. The theoretical summary of the wave equations expressed to the electric and magnetic field using the Finite Element Method (FEM) was approached as well, then, the deriving of the applied weak forms in time and frequency domain and their application methods was discussed and created to an electromagnetic simulation to represent the physical behavior of fields in gravity.

As seen as, the finite element time domain (FDTD) solving the Maxwell's equations in a general unstructured mesh has been presented in last time. The FDTD algorithm directly solves transient electric fields by applying Galerkin's method with homogeneous dirichlet boundary conditions to the electric field diffusion equation. To compute initial electric fields over an arbitrary conductivity earth model for a step-off source waveform, the secondary potential method is used to solve Poisson's equation. The method extends the successful1 use of edge elements in the frequency domain to the time domain in conjunction with face elements scheme for the time discretization.

Considering limitation of recent problem issue, in this paper, the Finite Element Method (FEM) will be used to solving homogeneous waveguide problems by approximated exterminations of functional to be higher accuracy than numerical methods presently employed, and reliably to product complete numerical sets at little computational cost. Its usefulness for solving problems in both convex and non-convex regions has been demonstrated by comparing computed results with analytically known solutions. It is shown how the FEM can be introduced to electrical engineering using only a few command of MATLAB program related directly to the FEM operating codes. The results of developed simulation shown that our program implemented using the Graphic User Interface (GUI) gives the proper results that can be used in education and solve any more complicated electromagnetic problems. This research illustrated the advantages of FEM compared to an analytical method and shown how the concepts acquired can be readily extended to other problem issues. However, the presented programming techniques can be employed for similar complex problems.

REFERENCES

(Arranged in the order of citation in the same fashion as the case of Footnotes.)

[1] Evan, L. and Davidson, D.B. (2010). "GPU-accelerated method of moments by example: Monostatic scattering". IEEE Antennas and Propagation Magazine, 52(6), pp. 120-135.

[2] Aodsup, K. and Kulworawanichpong, T. (2012). "Simulation of Lightning Surge Propagation in Transmission Lines Using the FDTD Method". World Academy of Science, Engineering and Technology.

[3] Cakir, G., Sevgi, L., and Ufimtsev, P.Y. (2012). "FDTD modeling of electromagnetic wave scattering from a wedge with perfectly reflecting boundaries: Comparisons against analytical models and calibration". IEEE Transactions on Antennas and Propagation, 60(7), pp. 3336-3342.

[4] Francés, J., Pérez-Molina, M., Bleda, S., Fernández, E., Neipp, C., and Beléndez, A. (2012). "Educational Software for Interference and Optical Diffraction Analysis in Fresnel and Fraunhofer Regions Based on MATLAB GUIs and the FDTD Method". Education, IEEE Transactions on, 55(1), pp. 118-125.

[5] Gaffar, M. and Jiao, D. (2014). "An explicit and unconditionally stable FDTD method for electromagnetic analysis". IEEE Transactions on Microwave Theory and Techniques, 62(11), pp. 2538-2550.

[6] Choocadee, S. and Akatimagool, S. (2010). "Development of efficiency em simulation tool for capacitive and inductive obstacle analysis". Electrical Engineering/Electronics Computer Telecommunications and Information Technology (ECTI-CON), 2010 International Conference.

[7] Zimmerman, and William, B.J. (2006). "Multiphysics modeling with finite element methods". Vol. 18, World Scientific Publishing Co., Inc.

[8] Zhong, L. and et al. (2012). "Convergence and optimality of adaptive edge finite element methods for time-harmonic Maxwell equations". Mathematics of Computation, 81(278), pp. 623-642.

[9] Um, E.S., Harris, J.M., and Alumbaugh, D.L. (2012). "An iterative finite element time-domain method for simulating three-dimensional electromagnetic diffusion in earth". Geophysical Journal International, 190(2), pp. 871-886.